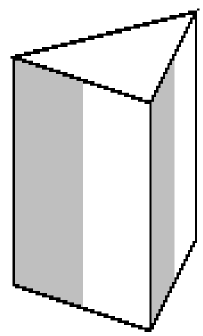
Exam Symmetry in Physics

Date	June 28, 2022
Time	8:30 - 10:30
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- All subquestions (a, b, c) of the three exercises have equal weight
- Illegible answers will not be graded
- Good luck!

Exercise 1

Consider the triangular block depicted in the figure below:



The base is an equilateral triangle. The rectangular sides all have the exact same pattern where the left half is grey and the right half is white.

(a) Identify all symmetry transformations (rotations and reflections) that leave this object invariant and call the group that they form G_{TB}. Show that G_{TB} is not isomorphic to the dihedral group D_3, but of the same order.

(b) Construct the character table of G_{TB} and explain how the entries are obtained. Indicate which irreps are faithful.

(c) Show whether G_{TB} allows for an invariant vector, an invariant axial vector or neither. Explain how the answer can be understood from the figure and its symmetry transformations.

Exercise 2

Consider the group of real orthogonal 3×3 matrices $O(3)$.

(a) Show explicitly that the Kornecker δ_{ij} $(i, j = 1, 2, 3)$ is invariant under $O(3)$ transformations.

(b) Show that all reflections in R^3 can be written as minus the 3×3 identity matrix times a rotation and show this explicitly for a reflection in a two-dimensional plane in R^3.

(c) Explain that $\vec{u} \cdot (\vec{v} \times \vec{w})$ is a pseudoscalar, i.e. describe how it behaves under *all* $O(3)$ transformations.

Exercise 3

Consider the Euclidean group $E(2)$ of isometries in two dimensions.

(a) Provide the definition of isometry and give the 4 non-trivial types of isometries in two dimensions.

(b) If we denote the elements of $E(2)$ by $(O|\vec{a})$, where $O \in O(2)$ and $\vec{a}$ denotes a translation over a two-vector $\vec{a}$, then show (for instance by acting on a two-vector) that the composition law of $E(2)$ is: $(O_1|\vec{a}_1)(O_2|\vec{a}_2) = (O_1 O_2|O_1\vec{a}_2 + \vec{a}_1)$.

(c) Provide a 3-dimensional matrix representation of $E(2)$ and explain why a 2-dimensional matrix representation is not possible for this group.